STATISTICAL FATIGUE AND RESIDUAL STRENGTH ANALYSIS OF NEW/AGING AIRCRAFT STRUCTURE

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Abstract

The structural integrity of both military and civilian transport aircraft fleets is governed by Department of Defense and Federal Aviation Administration regulations, respectively, dictating use of fracture mechanics techniques to determine the durability and damage tolerance of the aircraft structure. Driven by economic pressures, both fighter and transport aircraft are remaining in service longer than their original design lifetimes. Maintaining mission readiness and ensuring the safety of the flying public is of utmost concern and the responsibility of the aircraft manufacturers, aircraft operators, and airworthiness authorities. In general, a fracture mechanics based crack growth prediction model is just another tool in the designer and maintainer's toolbox to safely and efficiently manufacture and operate the aircraft. Furthermore, state-of-the-art fatigue life prediction algorithms can only consider simple, well-behaved structural cracking problems. Unfortunately, all modern aircraft are complex assemblies with diverse materials and joining methods. As a result, mission planners and combat leaders are forced to maintain the old paradigm of over-conservative fatigue life predictions and *find-it* and *fix-it* corrosion control programs. As a result, the main outcome of our Challenge Project is better fleet management through more robust fatigue life predictions in both peace- and war-time.

Description of Our Challenge project

This Challenge Project has three main tasks focused on fatigue crack growth and residual strength prediction. Specifically, task 1 efforts increase the accuracy of the stress intensity factor solutions (K) used in fatigue crack growth predictions. Incidently, it is worthwhile to note that the __K-solutions developed will apply to all aircraft in both the military and civilian fleets__. Methods developed in task 2 will allow the mission planner to assess the level of risk associated with future missions with respect to how damage accumulates per flight. Furthermore, life prediction codes have not considered environmental attack and task 3 considers the effect of environmental degradation on structural integrity. This paper only considers task one efforts as this is the first year of our Challenge Project.

In fatigue analysis using fracture mechanics, tabulated data for K solutions are commonly used. However, available databases cover mainly single cracks for a rather restricted parameter set and very simple geometries. In the case of diametrically opposed corner cracks at a hole of different sizes and of complex shapes, the situation is worse. Figure 1 shows a 10 inch crack in an aircraft exemplifying the type of problem of interest. In the case of three or more cracks, a data base with K's for realistic parameter sets would become prohibitively large. By using the mathematical splitting scheme described in Ref. [1] K's can be effectively calculated as a part of the fatigue crack growth analysis even in the case of multiple-crack configurations, Ref. [2].

A system for analysis of multiple crack fatigue growth was designed and tested on large cluster computers made available through the US DoD High Performance Computing Modernization Program. The present project has the objective to demonstrate that these types of computations can be performed effectively and reliably on large computer systems having thousands of processors hence cutting computer time down to a minimum and thereby making advanced analysis of this type practically feasible.

The Splitting Method

The splitting scheme was the basic mathematical method used in the project, see Ref [1]. The method can be used to efficiently and reliably calculate the thousands of K-solutions needed for the various crack sizes needed in a single fatigue analysis, or the millions of K-solutions needed in a statistical fatigue analysis, see Ref. [2]. A solid mathematical foundation is given in Ref. [1].

In the splitting method, the problem is split into subproblems. The simple multi-site cracking scenario shown in Figure 2 (left part) is used to show the decomposition of the problem of interest into global *un-cracked* problems and local problems with a single crack.

The three problems a, b, c (analysis levels I, III and V, respectively in Figure 4) are:

a. Global Crack Free Problem: The solution of the global crack free problem is $U_G^{(0)}$. This sub-problem (i.e the full aircraft problem) is independent of the number and size of cracks under consideration, hence this very time consuming problem need be solved only once.

b. A Set of M Local Problems: A local model is developed for each crack size parameter and contact surface parameter, determined iteratively as a part of the global nonlinear solution. The applied load consists of L different normalized crack surface tractions with the solutions denoted as $\left\{ U_L^{(m,l)} \middle| m = 1, 2 \ldots M, l = 1, 2, \ldots L \right\}$. The local models contain a single, perhaps large, crack of complex shape

c. A Set of Global Crack Free Problems: The global model in Figure 2c is analyzed for prescribed *jumps* in tractions and displacements at the surfaces Γ_i used in the local problems. The solutions are denoted $\left\{ U_G^{(m,l)} \middle| m = 1, 2 \ldots M, l = 1, 2, \ldots L \right\}$.

The approximate solution $\overline{U}$ to the exact 3D solution U is written as

$$\overline{U} = U_G^{(0)} + \sum_{m=1}^{M}\sum_{l=1}^{L} \alpha_{(m,l)} U_G^{(m,l)} - \sum_{m=1}^{M}\sum_{l=1}^{L} \alpha_{(m,l)} U_L^{(m,l)} \qquad (1)$$

where $\alpha_{(m,l)}$ are scaling factors determined by solving a small set of linear equations. The linear equations are obtained from the condition that crack surfaces shall be traction free. Thus with the known $U_G^{(0)}, U_G^{(m,l)}$, and $U_L^{(m,l)}$, the solution, $\overline{U}$ can be calculated with *virtually no computational cost per crack configuration*. This is very important when numerous solutions to the global problem are needed for different crack patterns. The computational efficiency of the strategy devised makes it feasible to perform Monte Carlo type studies of three dimensional (3D) multiple-site fatigue crack growth problems. We note that as L goes to infinity Eqn. (1) is exact for arbitrary large non-planar cracks surfaces.

Figure 3a shows a small part of a fuselage shell analyzed in tasks 2 and 3 of the project. The sub-mesh shown has 370 rivets and numerous possible 3D cracks. Figure 3b shows a local problem, of about 1000 problems considered, in a characteristic study. Shells and rivets are modeled as three-dimensional objects. Note that fatigue crack growth of one, or, many (hundreds) of interacting cracks can be analyzed using the splitting scheme.

The System Setup
Figure 4 shows a flowchart of the numerical implementation of the mathematical scheme. The finite element program STRIPE used at levels I, III, V uses a *hp*-version of the finite element method and various novel mathematical methods to achieve scalability and computational efficiency. The costly part of the solution is to derive the local solutions $U_L^{(m,l)}$ (95% of cost) and the global solutions $U_G^{(m,l)}$ (5 % of cost), respectively (Eqn 1) during task 1. Computational characteristics of the solution steps I, III and V are now briefly summarized.

Local Solutions $U_L^{(m,l)}$
A typical computational domain is shown in Figure 3b. The contact problem between rivet-stiffeners-skin requires for its solution of the order of hundreds of thousands degrees of freedom for cases when a virtually exact solution is sought. Such computations are performed using typically 8 CPU's per local problem. Since local problems are uncoupled, almost perfect *scalability* is obtained.

Global Solutions $U_G^{(m,l)}$
For multiple site *fatigue* analysis the domain shown in Figure 3a is in most cases sufficiently large. A high accuracy solution has about 10^7 *DoFs*. The number of *DoFs* in the largest models analysed in the project has $>10^8$ *DoFs*. FE-models of this size allow a virtually exact numerical solution for arbitrary 3D crack patterns being derived as a part of the fatigue analysis. We remark that the largest models (with up to 10^9 *DoFs*) are designed for statistical residual strength analysis of fuselage and wing sections with multiple-site damage during tasks 2 and 3 in the Challenge Project.

The Approximate Solution $\overline{U}$

The approximate solution $\overline{U}$ is obtained from Eqn. (1). In fatigue analysis, Eqn. (1) is used repeatedly where different solutions $U_G^{(m,l)}$ and $U_L^{(m,l)}$ corresponding to actual crack sizes and crack locations at time t are used. Such computations are performed on level VII, with negligible computational cost, and can be done in parallel for different crack scenario (statistical fatigue analysis).

Accurate *K*-Calculation in Case of Very Large Irregular Cracks in Complex Domains

The primary unknowns in the splitting scheme for calculation of K's are normal and shear tractions T_1, T_2, T_3 acting on crack faces on small sub-problems. For cracks which are small, or of same size as local geometry dimensions (for example a hole diameter), the unknown tractions T_k can be well approximated by polynomials in the form,

$$T_k(r,z) \approx \sum_{i=0}^{p-1} \sum_{j=0}^{p-1} t_{ij}^{(k)} r^i z^j \qquad (2)$$

where $t_{ij}^{(k)}$ are unknowns in the discrete version of the splitting scheme *(r,z)* are coordinates in the (possibly curved) crack plane. For cases when cracks are very large compared to local structural dimensions, the crack surface stress distributions cannot be well approximated by low order polynomials (p = 2-6, for example) in the form of Eqn. (2). Figures 1 and 5 shows a typical case where a relatively small part of a crack face is located in a region with a countersunk hole. Figure 6 shows a typical mesh for the hp-version of FEM. For some parameter sets of interest the hole radius is very small compared to the crack size. For such cases, the stress intensity function $K(\varphi)$ will be relatively smooth along the main part of the crack front and will blow up in a region near the countersunk hole. Figure 7, which shows the stress intensity function $K_I(\varphi)$ for remote tensile loading shows this blowup.

In order to efficiently analyze problems with very large cracks, or problems with (large) irregular crack faces, a new version of the splitting method had to be developed. The basic idea is to approximate tractions $T_k(r,z)$ by piecewise polynomials defined on a one-dimensional radial "mesh". This "mesh" is defined by the radices $r_j, j = 0,1,\ldots$. The new traction approximation in case of very long cracks of irregular shape is,

$$T_k^{(hp)}(r,z) \approx \sum_{m=0}^{M} \sum_{n=0}^{p-1} v_{mn}^{(k)} N_m(r) z^n \qquad (3)$$

where $v_{mn}^{(k)}$ are coefficients to be determined.

The functions $N_m(r)$ are polynomials, nonzero only in a local *r*-interval, as in the case of the standard *hp*-version of the finite element method. Hence, the functions $N_m(r)$ are identical to polynomials $P_m(r)$ defined on local *r*-intervals.

The practical implementation of the *hp*-version of the splitting method is relatively straight forward. The steps indicated in Ref. [2] remains in principle the same, one difference is that the numerical integration has to be carried out over radial sections of the crack surface where normal and shear tractions are applied in the finite element analysis (local problems). Another difference is that some coefficient matrices needed have to be assembled in a way similar to the finite element analysis procedure. The finite element mesh of the local domain, Figure 5, must also have element edges corresponding to radices $r = r_j, j = 0,1,\ldots$ in order to apply traction loading. Radial elements, for the *hp*-splitting scheme analysis, with a grading factor 4 is used in the analysis, that is: $(r_{j+1} - r_j)/(r_j - r_{j-1}) = 4, j = 1,2,3$. Convergence tests on configurations with $c/R \approx 100$ shows that the relative pointwise error in $K_I(\varphi)$ can generally be made lower than 10^{-3} using polynomial order $p = 5$ of the functions $N_m(r)$ and meshes of the type shown in Figure 5. Figure 7 illustrates that the $K_I(\varphi)$-solutions for polynomial orders p = 4, 5, 6 are inseparable with the actual resolution. In other words, the numerical solution is converged for a polynomial order $p = 4$.

Creation of Worlds Largest *K*-Data Base For Fatigue and Residual Strength Analysis

A stress intensity factor data base applicable to single and multiple cracks in basic structural elements is being created in step1 of the Challenge Project. The data base will be implemented in the crack growth program *AFGROW* developed by the USAF. The number of crack geometries for which stress intensity factors are calculated are too numerous to list;

however, they can be categorized as shown in Table 1. Figure 8 shows some of the crack geometries studied in case of a plate with a single hole.

Table 1. *K* Solution Parameter Space

Local Geometry	Hole Type		Crack Type		Loading			Number of *K* Solutions (Millions)
	Countersunk	Straight Shank	Part-Through the thickness	Through the thickness	Tension	Bending	Bearing	
Plate with Single Hole	1	1	1	1	1	1	1	29.34
Plate with Two Holes	1	1	1	1	1	1	1	38.68
Lug with Single Hole	1	0	1	1	1	1	1	4.99

The crack geometry is defined in terms of the local and crack dimensions; plate width, *W*, plate thickness, *t*, hole radius, *r*, hole diameter, *D*, countersink angle, θ, countersink depth, *B*, ligament spacing, *L*, crack depth, *a*, and crack length, *c* where,

> Plate Geometry
>
> r/t = 0.075, 0.1, 0.2, 0.333, 0.5, 1.0, 2.0, 3.0, 6.0
> L = 1D, 2D, 3D, 4D where D is max(D_1, D_2) applies to two hole cases only
> D_1/D_2 = 0.5, 1.0, 2.0 applies to two hole cases only
> B = 1D_2, 2D_2, 3D_2, 4D_2
>
> Part-Through Crack Geometry
>
> a_i/c_i = 0.1, 0.125, 0.1667, 0.25, 0.5, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0
> a_i/t = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95
>
> Through Crack Geometry
>
> a_i/c_i = 0.1, 0.125, 0.1667, 0.25, 0.5, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0
> a_i/t = 1.05, 1.5, 2, 3, 4, 5, 10

Hence, the structurally most significant crack shapes and loading conditions are considered. Table 1 shows for example that 39 million geometries are needed in order to cover the parameter set for the relatively simple twin hole geometry with 1-4 cracks. The splitting scheme described above is used to analyze the 73 million crack geometries with a guaranteed relative error less than 10^{-2} in stress intensity factors K_I, K_{II} and K_{III} along the entire crack front for a number of loading cases. Figure 13 shows the principles behind mesh design for the *hp*-version of FEM which is considered in the design of the 86000 single-crack meshes needed, Ref. [2].

Production Capacity oftThe ASC/ERDC MSRC's June 2006 for Task 1 Problems

The execution of 86,000 level III jobs and 4,000 level V jobs (Figure 4) using 4-32 processors within a 2-8 hour period constitutes a real challenge to the MSRC's centers utilized in the present Challenge Project. Except for the computational burden (4.4 million CPU-hours needed during 2006), the job control language used to optimize the job environment have repeatedly been found to malfunction or simply fail to cope with the many thousand level III-jobs running/queued. Queing thousands of relatively small jobs also constitutes a disadvantage for HPC-users asking for very many processors in single jobs since the LSF continuously tends to reject larger jobs in favor of smaller jobs. In order to avoid this drawback, a limitation on the maximum number of users per job has been enforced at ERDC and NAVO. This limitation initially led to an unacceptably low production capacity in the present project. On the SGI/O3k and SGI/Altix hardware at the ASC/ERDC-centers a technique was developed where large CPU-sets are requested and many smaller 8 processors jobs are executed in parallel, in so-called pipes, inside the large CPU-sets. In order to further avoid shortcomings of the job controllers, a number of jobs are queued in each pipe. This technique also has disadvantages since frequently a few jobs in the CPU-set may take 3-8 times longer execution time compared to being executed in single-job mode. The reason for such stalling behavior remains unknown despite significant efforts made in the project supported by SGI-specialists.

Figure 9 shows the throughput observed when executing 1,500 III-jobs and 60 level V-jobs during the time period June 1-3 on the four hardware systems ASC/hpc11, ASC/eagle, ERDC/ruby and ERDC/emerald, respectively. Due to the different capacities of the four systems used in the test, the simplest load balancing algoritm was applied by assigning 800/400/200/100 level III jobs and 32/16/0/12 level V-jobs on eagle/hpc11/emerald and ruby-systems, respectively. The level III-jobs, which were submitted under a 10 minute period, were executed in 4 parallel pipes using 32 processor CPU-sets. Since the finish time of the level III jobs could not be anticipated, the level V jobs, running in parallel, were not submitted as soon as they could have been. The effect is small as the level V jobs are roughly 5% of the total computational effort in this benchmark. The computational work in the test corresponds to 1.5% of the total

computational work needed in task 1 of the Challenge Project. The total wall-time needed (Figure 9) was less than two days indicating that all 86,000 level III jobs and 4000 level V-jobs could be analysed during an 80 days period.

Results Achieved the First 9 Months in the Challenge Project

The main results obtained during task 1 of this Challenge Project are:

- Development of a so-called mathematical splitting scheme with error control for fatigue and residual strength analysis of complex built-up structures with multiple large irregular cracks described in the present paper

- Development of 14 mesh generators for levels III and I/V respectively

- Implementation, optimisation, adaption of the splitting scheme to systems eagle/hpc11/ruby/emerald and NRL/morpheus-systems, respectively. Creation of production system (Figure 8)

- Derivation and delivery of the first 8 million K-solutions to Air Force Research Laboratory (AFRL)

- Development of a new direct solver with much improved scalability properties for level V analysis to be used in the next phases of the Challenge Project

Scalability of hp-version FE-Solver Used

In the next phase of the Challenge Project, truly large-scale structures will be analysed at level V as a part of the residual strength analysis of the C-130 wing. The main computational work hence will shift from level III dominated to level V dominated. In order to to perform truly large scale analysis with, say 100,000 right hand sides, a highly scalable direct solver is by far the most efficient alternative. A solver for the hp-version of the finite element method that uses mixed multi-level MPI and OpenMP has been developed the last year. Figure 12 shows that using this solver the splitting scheme scales perfectly during the analysis of the local problems (level III). The global problem (level V) did not scale very well above 256 CPU's due to extensive I/O.

Conclusion

Significant progress has been made toward increasing the robustness of fatigue life predictions of aircraft structure during the first year of this Challenge Project. With all the mesh generators and productions system complete for calculating the 73 million K solutions in addition to the new direct solver for the large scale analysis, year two will focus on productions run to complete the remaining 65 million K solutions and residual strength analysis of non-environmentally degraded C-130 center wing box structure.

Acknowledgents

REFERENCES

[1] Babuska I., Andersson B., The Splitting method as a tool for multiple damage analysis, SIAM J Sci Comp, Vol 26, No 4, 2005, pp 1114-1145.

[2] Fawaz S. A., Andersson B., Accurate stress intensity factor solutions for corner cracks at a hole, Engineering Fracture Mechanics 71, (2004), pp 1235-1254.
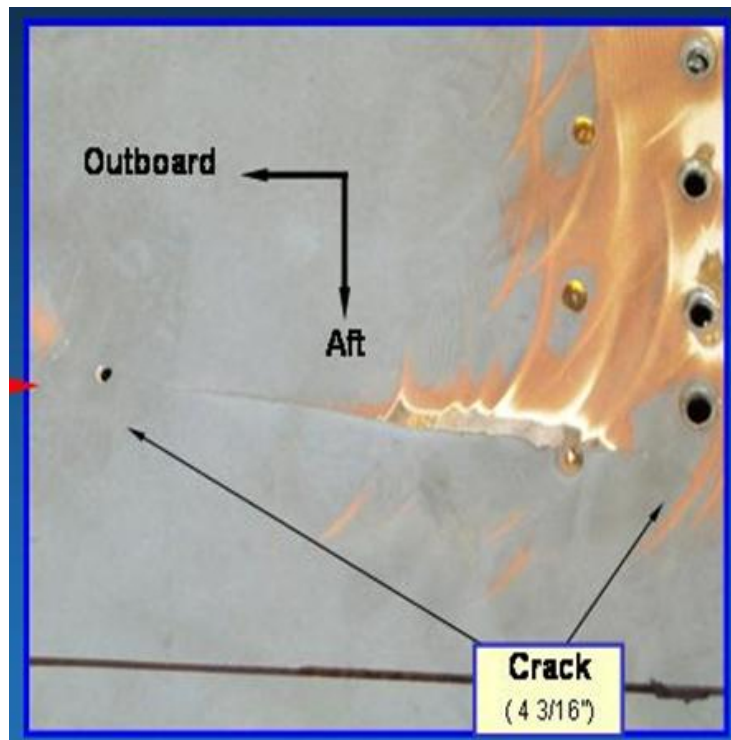
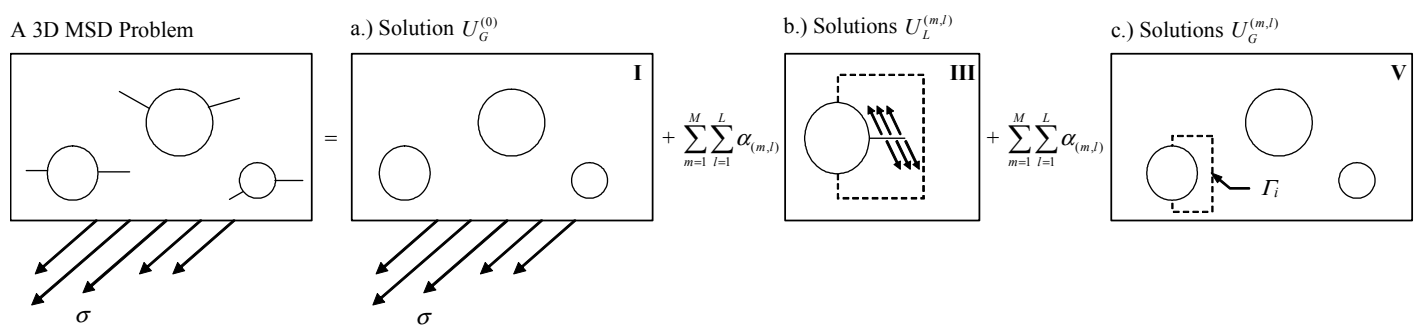Figure 1 Crack scenario of interest in the Challenge Project

Figure 2 The three splitting scheme sub-problems labeled, I, III and V respectively
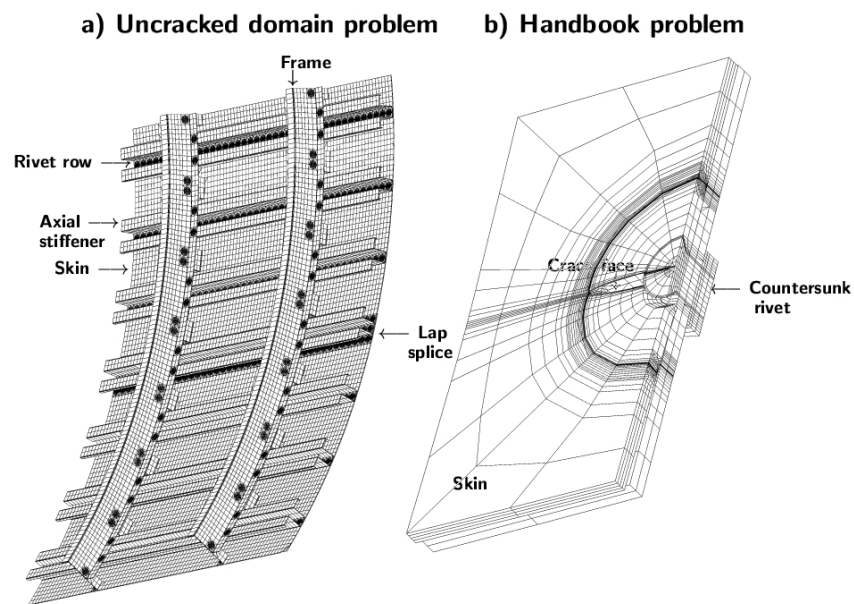
Figure 3. Part of fuselage shell without cracks (compare sub-problem a in figure 2) and a typical local domain near a lap-splice with a 3D crack. Unknowns in the statistical nonlinear analysis are contact surfaces between the countersunk rivets and the plate
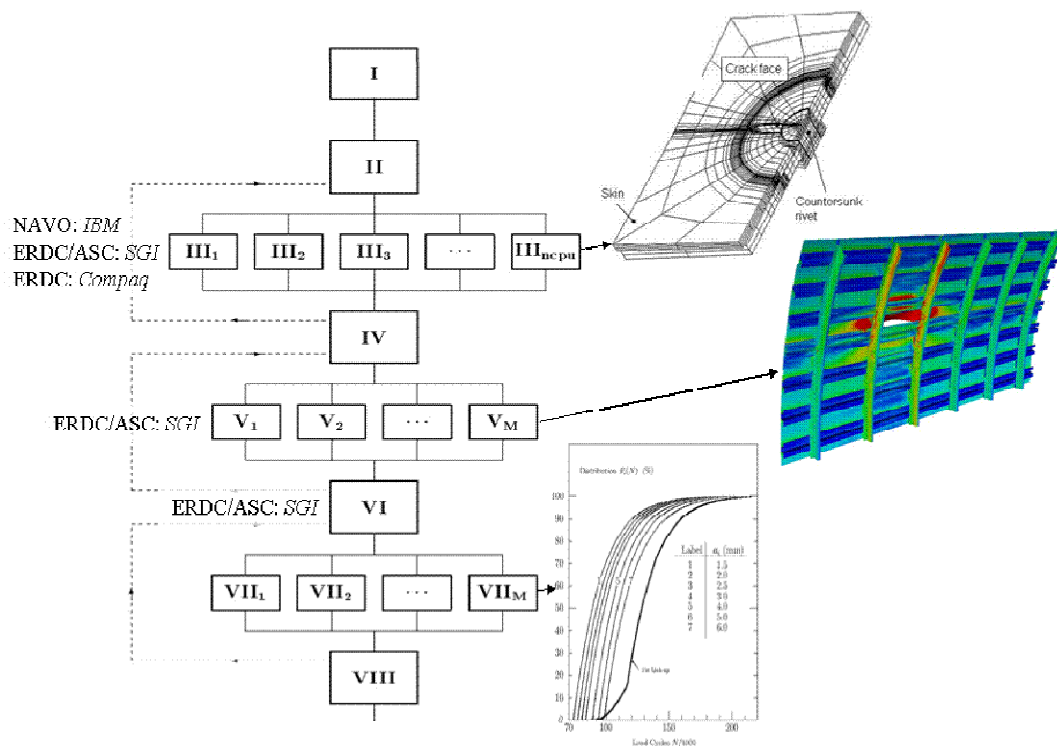

Figure 4. Schematic picture of splitting scheme for statistical analysis of built-up aircraft structures. The figure shows schematically the systems setup for large scale fatigue analysis. The local problems with the single crack (level III problems) can be solved completely in parallel, hence, the larger the computer, the smaller the time needed to study all (thousands) crack geometries needed. The level V job being the analysis of the complex global problems are the crucial part what regards taking advantage of having access to a large computer with many CPU's.
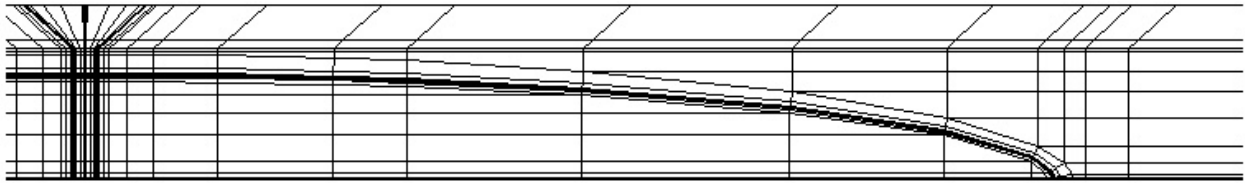
Figure 5. Part of mesh for *hp*-version of FEM and splitting method for large crack case with parameters *R/h* = 0.075, *a/h* = 0.6, *c/a* = 10, *c/R* = 80. *R* is radius, *h* is plate thickness, *c* and *a* size of half ellipse axes

b) Torus with quadratic crossection enclosing elliptical crack front

Vertex a
Crack Front
Vertex c

a) FE-mesh, b/h=0.25, a/t=0.1, c/a=10

d) Two toruses with circular crossections enclosing hp-mesh at crack front
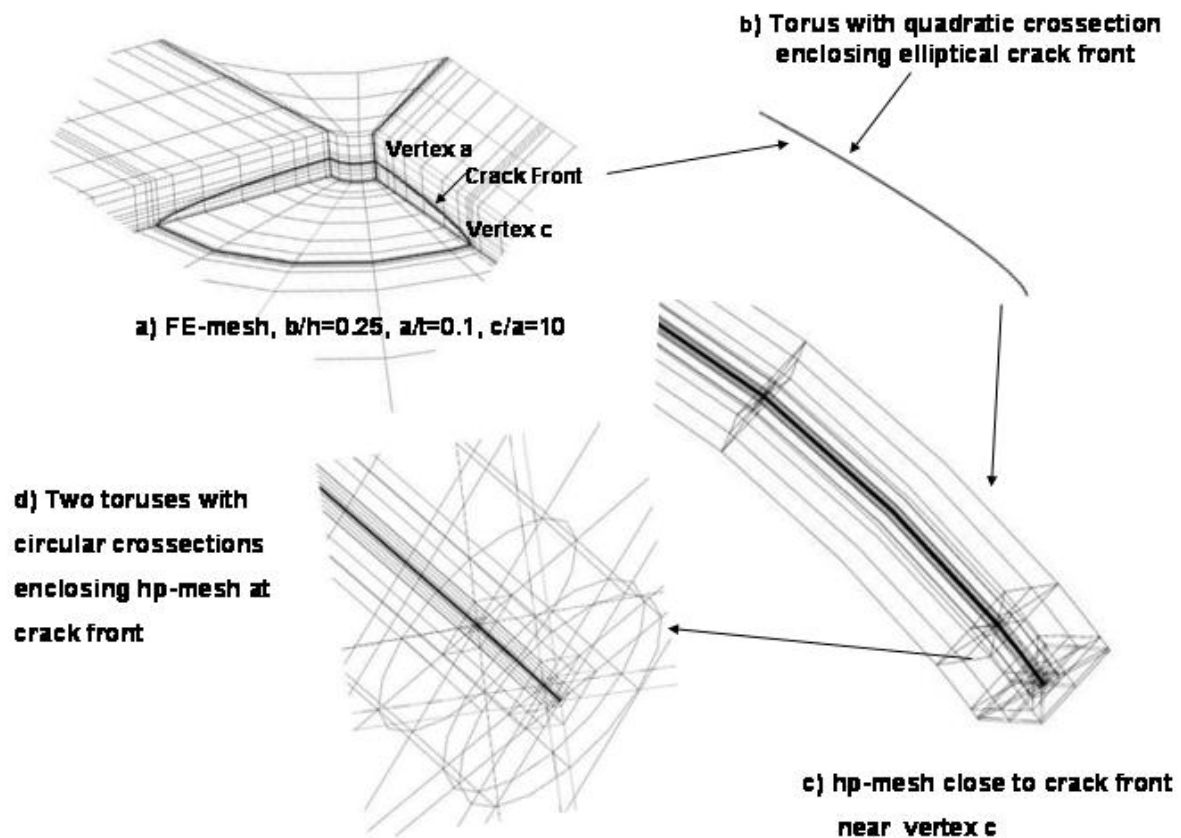
c) hp-mesh close to crack front near vertex c

Figure 6. Meshes for the *hp*-version of FEM designed to capture the *K* distribution along the entire crack front, including the vertices. For *c/R* very large a special version of the splitting scheme is used

Stress Intensity Function $K_I(\varphi)$

(plot: vertical axis labeled from 0.0 to 24.0 in increments; horizontal axis $2 \cdot \varphi/\pi$ from 0 to 1.0)

Inset geometry labels: $h = 16.667$, $a = 10$, $\varphi=0$, $R =1.25$, $c =100$

Three finite element solutions for polynomial orders p=4,5,6, respectively.
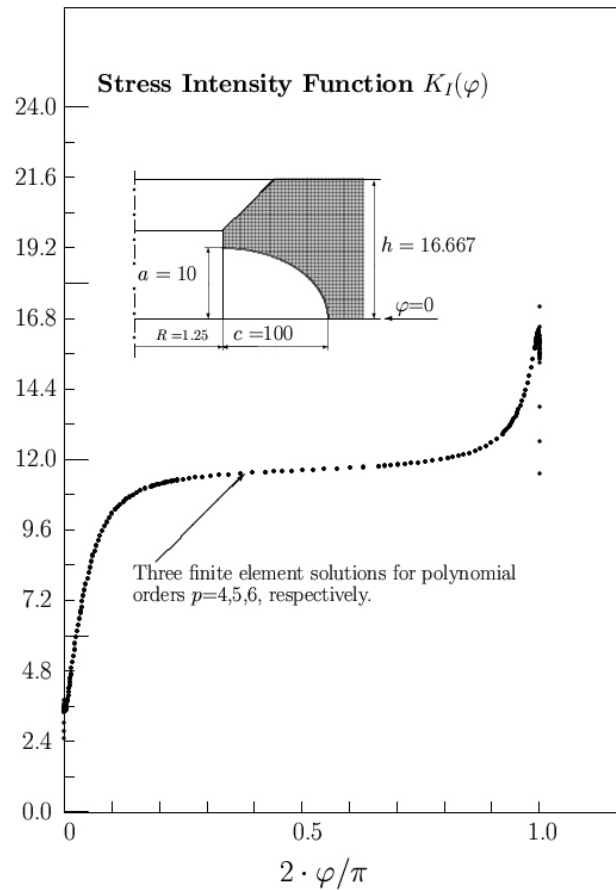
$$2 \cdot \varphi/\pi$$

Figure 7. Variation of stress intensity factor $K_I(\varphi)$ for double crack at countersunk hole in plate subject to remote uniform tensile loading σ = 1. Geometry parameters are R/h = 0.075, a/h = 0.6, c/a = 10, c/R = 80 with R = 1.25. The three solutions shown are obtained using the *hp*-version of the splitting scheme where tractions are approximated by polynomials of order p = 4,5,6, respectively. The three solutions are inseparable with actual resolution

Flowchart Production System

Input: Case No and R/t

(Automatic) identification of actual parameters:
- Cases 1-5 : a/t, a/c (manual input b/t)
- Cases 6-8 : a/c, $2C/t$
- Cases 9-32: a/t,a/c (manual input $D1/D2$, L and B)
- Cases 33-36: a/t, a/c, W/D, e/D

History Data Base

1 Global FE-mesh → Loop M and N — I

Optimal M, N

100'eds of local FE-meshes — II

III_1 III_2 III_3 $\cdots$ III_{ncpu}

IV

Calc. error — $V_1^\bullet$ $V_2^\bullet$ $\cdots$ $V_M^\bullet$

Single crack data base for converg. test — VI

VII_1 VII_2 $\cdots$ VII_L

K-data Base (this case) — VIII

— Exit

(local mesh labels: MTC1, MRTC2, MPTC1, MTSS1, MTSS2, MTLS, MNT1, MFT1, MNTSS, MNTSS2, MPTL1)
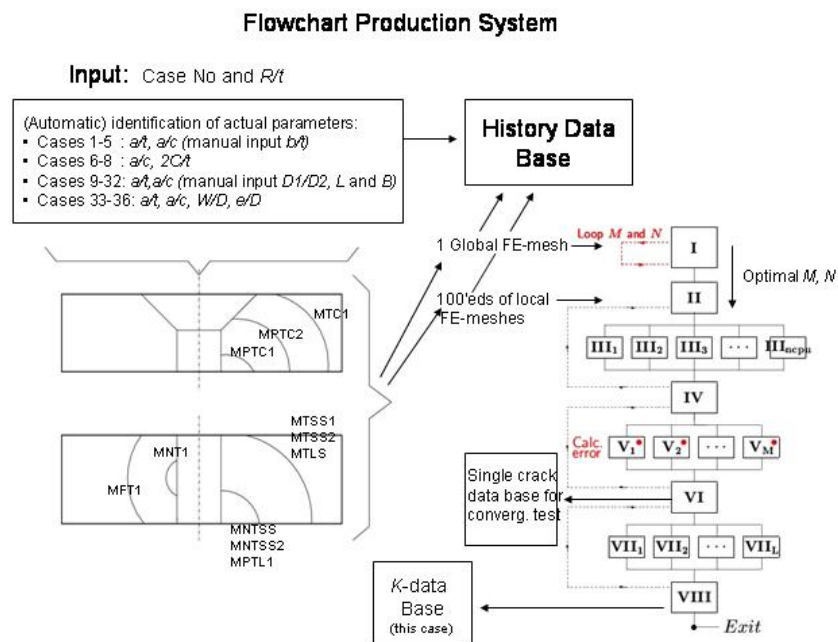
Figure 8. Flowchart of system for analysis of 73 million crack configurations. A history data base is created in order to be able to trace the underlying data used.
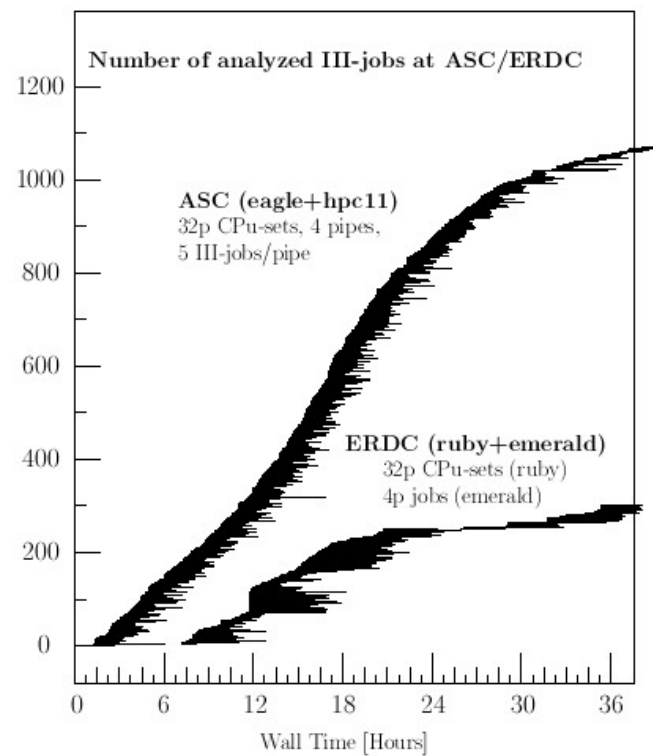
Figure 9. Throughput on the four platforms ASC/Eagle, ASC/hpc11, ERDC/ruby and ERDC/emerald during the period June 1-3 2006.
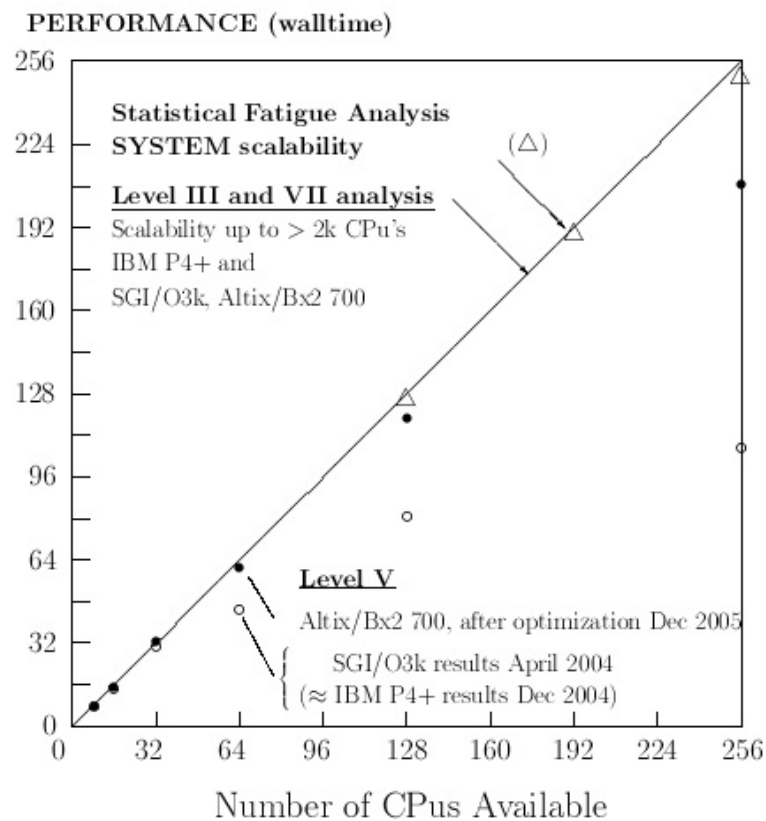

Figure 10. Scaling properties of the splitting scheme during task 1 on different hardware platform