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Finite Element Analysis of a Pipe Elbow Weldment Creep-Fracture Problem Using an Extremely Accurate 27-Node Tri-Quadratic Shell and Solid Element Formulation

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Abstract

In most finite-element-analysis codes, accuracy is achieved through the use of the hexahedron hexa-20 elements (a node at each of the 8 corners and 12 edges of a brick element). Unfortunately, without an additional node in the center of each of the element's 6 faces, nor in the center of the hexa, the hexa-20 elements are not fully quadratic such that its truncation error remains at h(0)^2, the same as the error of a hexa-8 element formulation. The symbol (^) denotes raising the preceding quantity to a power of the subsequent quantity. To achieve an accuracy with a truncation error of h(0)^3, we need the fully-quadratic hexa-27 formulation. A competitor of the hexa-27 element in the early days was the so-called serendipity cubic hexa-32 solid elements (see Ahmad, Irons, and Zienkiewicz, Int. J. Numer. Methods in Eng., 2:419-451 (1970)). The hexa-32 elements, unfortunately, also suffer from the same lack of accuracy syndrome as the hexa20's. In this paper, we investigate the applicability of the fully quadratic hexa-27 elements to three problems of interest to the pressure vessels and piping community: (1) The shell-element-based analysis of a barrel vault. (2) The solid-element-based analysis of a welded pipe elbow with a longitudinal surface crack in one of its weldments. (3) The solid-element-based analysis of the elastic bending of a simple cantilever beam. Significance of the highly accurate hexa-27 formulation and a comparison of its results with similar solutions using ABAQUS hexa-8, and hexa-20 elements, are presented and discussed.

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1. Introduction

Recent experimental results (see Fig. 1) on the creep-fracture damage with minimum time to failure (MTTF) varying as the 9th power of stress (see, e.g., Cohn, Cronin, Faham, Bosko, and Liebl [1], and NRIM [2]), and a theoretical derivation of the consequence that the coefficient of variation (CV) of MTTF is necessarily 9 times that of the CV of the stress (see Fong, et al. [3]), created a new engineering requirement that the finite element analysis of pressure vessel and piping systems (PVPS) in power-generation and chemical plants be more accurate with perhaps a less than one percent of error when dealing with a leak-before-break scenario. This requirement becomes critical, for example, when a small leakage is found in the vicinity of a hot steam piping weldment next to an elbow, and a risk-informed decision to repair before the scheduled shutdown is needed.

As shown in [1-3], the creep rupture time vs. stress relationship is modeled by a power law (see Eq. (1) below):

$$t = \lambda \sigma^{C}, \tag{1}$$

where t = creep rupture time (hour), σ = stress (MPa), λ and C are constants to be determined from experimental data.

In most cases of engineering design, the stress σ is estimated by a finite element analysis. As shown by Fong, et al. [3], an uncertainty in the estimate of the stress can lead to an uncertainty in the estimate of the creep rupture time. To quantify this relationship, let us define $m(\sigma)$ and $sd(\sigma)$ as the mean and standard deviaton of the stress, σ, and $m(t)$ and $sd(t)$ as the mean and standard deviation of the creep rupture time, t . Let us further define the coefficient of variation of stress, $cv(\sigma)$, by $cv(\sigma) = sd(\sigma) / m(\sigma)$, and the coefficient of variation of the creep rupture time, $cv(t)$, by $cv(t) = sd(t) / m(t)$. An application of the classical law of error propagation (see Ku [4]) to Eq. (1) yields the following simple result:

$$cv(t) = |C| * cv(\sigma). \tag{2}$$

This means we need to assess carefully the uncertainty of the stress estimate, because a 1 % coefficient of variation of stress can lead to a 9 % coefficient of variation of the creep rupture time.

In most finite-element-analysis codes, accuracy is achieved through the use of the hexahedron hexa-20 elements (a node at each of the 8 corners and 12 edges of a brick element). Unfortunately, without an additional node in the center of each of the element's 6 faces, nor in the center of the hexa, the hexa-20 elements are not fully quadratic such that its truncation error remains at $h(0)^2$, the same as the error of a hexa-8 element formulation (see Zienkiewicz and Taylor [5]).

To achieve an accuracy with a truncation error of $h(0)^3$, we need thTee fully-quadratic hexa-27 formulation. A competitor of the hexa-27 element in the early days was the so-called 'serendipity' cubic hexa-32 solid elements (see Ahmad, Irons, and Zienkiewicz [6]). The hexa-32 elements, unfortunately, also suffer from the same lack of accuracy syndrome as the hexa20's.

In a 3-part series of papers, of which this paper is Part I, we investigate the applicability of the fully quadratic hexa-27 elements to three problems of interest to the pressure vessels and piping community: (1) The shell-element-based analysis of a barrel vault. (2) The solid-element-based analysis of a welded pipe elbow with a longitudinal surface crack in one of its weldments. (3) The solid-element-based analysis of the elastic bending of a simple cantilever beam. The 3-part development of the theory is described below:

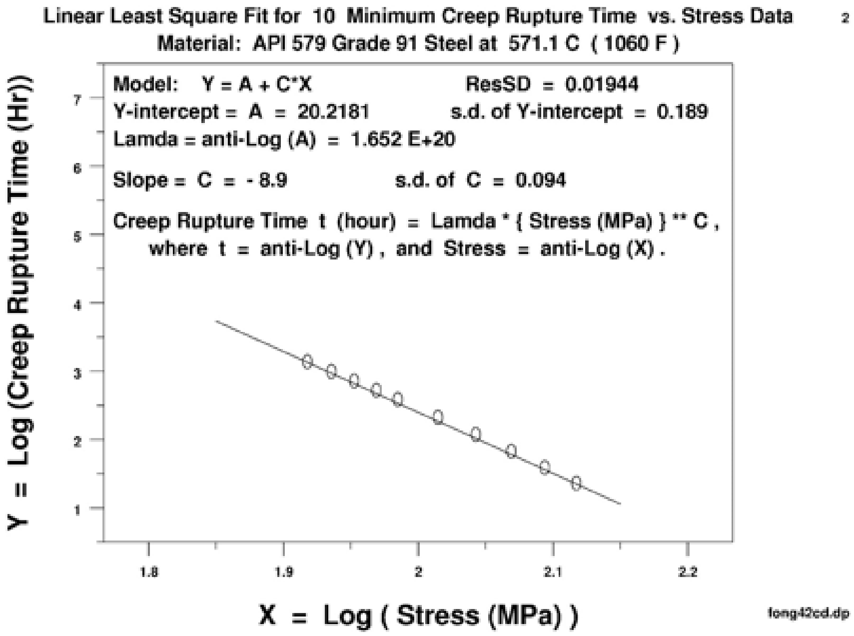

Fig. 1. A plot of Log (Creep Rupture Time) vs. Log (Stress) showing a slope of - 8.9 (after [1]).

Part I (this paper): We apply the mathematical theory of truncation errors [5, 9, 10] to prove that the 27-node element, Hexa-27, is superior to any of the commonly-used element, namely, Tetra-04, Hexa-08, and Hexa-20, because the truncation error of Hexa-27 is $h(0)^3$, and that of all the others are $h(0)^2$.

Part II (Fong, et al. [7]): In this part, we develop a method to estimate the uncertainty of a series of finite-element-mesh-density-parametric solutions (e.g., a specific stress component) as extrapolated by a 4-parameter logistic function, such that an upper bound of the stress can be inferred at infinite degrees of freedom.

Part III (Fong, et al. [8]): In this part, we develop a super-parametric approach to assessing the uncertainty of finite-element-based results not only by increasing the mesh density but also by varying both the element type and the solution platform.

Significance and limitations of this new approach to uncertainty quantification of the finite element method and some concluding remarks are presented. A list of references appear at the end of this paper.

2. The Hexa-27 Element and the Truncation Error Theory of the Finite Element Method

Most of the discussions on accuracy of the finite element method start with the completeness of the element representation using the adaptation of the Pascal Triangle [5], which in turn links back to errors of approximation by truncation of the terms in a binomial expansion. Here we explain the expansion in two dimensions using the Pascal triangle.

The expansion for a square (may be iso-parametric) is given by all the terms included by the diamond pattern as shown in Fig.2.

So for a quadratic square we have the following formula:

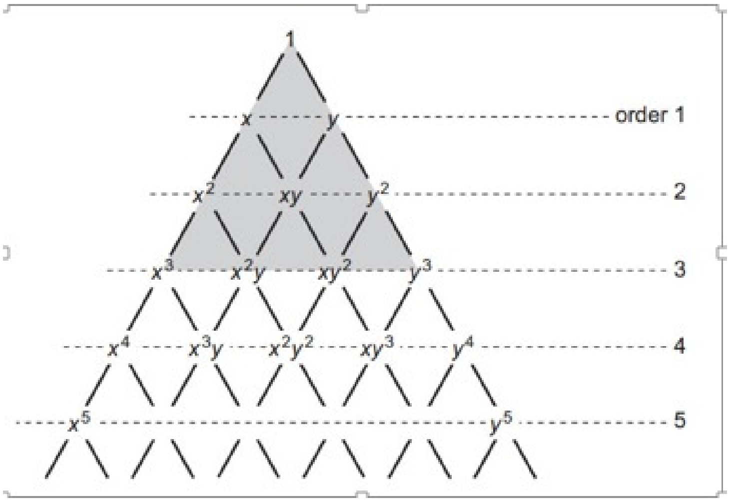

Fig. 2. The Pascal Triangle of the Order of Truncation Errors (after [5], Fig. 5.5).

$$u = [\,1,\ x,\ xy,\ y,\ x^2,\ x^2y,\ x^2y^2,\ xy^2,\ y^2\,]\ \{a\} + h(0)^3 \tag{3}$$

where u is a single degree of freedom in an element, {a} are nine undetermined coefficients, and $h(0)^3$ is the order of the truncation error.

The order of the truncation error can be read off from the Pascal Triangle as the lowest order that is not included in the Pascal Diamond for the square elements and by the triangle for simplex elements. By this we can write a similar expression for the quadratic triangle as

$$u = [\,1,\ x,\ xy,\ y,\ x^2,\ y^2\,]\ \{a\} + h(0)^2 \tag{4}$$

where {a} are six undetermined coefficients, and $h(0)^2$ is the order of the truncation error, e.g. the truncated terms x^2y, x^2y^2, xy^2 in (3) have been left out.

As shown by Zienkiewicz and Taylor [9], the so-called 'serendipity' elements are among the most popular of elements. In the two dimensional case these elements are obtained by dropping the x^2y^2 term so that the eight undetermined coefficients {a} result in a truncation error of $h(0)^2$ compared to that of $h(0)^3$ for the full nine terms.

The Pascal Triangle can be generalized to the third dimension by considering a third axis for z. Then the terms to be included or excluded are obtained by multiplying the terms in the third axis by the respective triangle or diamond for the first two axis as shown by the Fig. 2. All terms are obtained by cyclic permutation. As an example, we consider the case for the quadratic serendipity element in three directions. Here the excluded seven terms are

$$x^2y^2,\ y^2z^2,\ z^2x^2,\ x^2y^2z,\ xy^2z^2,\ x^2yz^2,\ x^2y^2z^2$$

This is for the 20-node hexa element with a truncation error of $h(0)^2$.

This may be compared to the full expansion quadratic element resulting in a 27-node hexa with a truncation error of $h(0)^3$. Though the value of the truncation errors are problem dependent, we can conclude qualitatively that the serendipity elements are a poor trade off because they result in a lower order of truncation error for the sake of approximately dropping of a quarter of the terms in the case of the 20-node hexa.

In the displacement method of the finite element analysis (FEA), we calculate the element stiffness and assemble these into a master stiffness. By the principle of virtual work, for an elastic body and using the usual finite element notation, we have

$$\{du\}' ([B] [N])' \; [D] [B] [N] \{u\} + d(h(0)^n)/dx_i \; D \; d(h(0)^n)/dx_i = \{du\}' \{P\} + \{dh(0)^n\}' \{P\} \qquad (5)$$

where ' indicates transpose of a vector or matrix,
[N] is the displacement to undetermined coefficient matrix,
[B] is the displacement to strain transformation matrix,
[D] is the strain to stress transformation matrix,
{P} is the load vector,
n is the truncation order of the error, and
$d(h(0)^n)/dx_i$ is the differentiation of the error term w.r.t. the three axis directions.

The inclusion of the error terms indicate the existence of the error even at convergence. It also shows that there are two types of error in the normal case, with both error terms contributing to the error in an integral sense. However in the case of strain singularities the differential local term dominates. This was noted by Zienkiewicz and Taylor [10] in their study of super-convergence. The error terms indicate that they are element dependent and with the orders as discussed previously.

The early work on modeling shells with solid elements was encouraging [6]. However, when it came to modeling thin shells, some doubt was cast on the ability of the solid elements to do so. There were three

reasons given for this. The first one was the apparently superior performance of the shell element formulation advanced in [6]. The second, based on the cubic solid elements used, was that this would lead to large computing times. The third was the worry that the thin shells would lead to numerical problems with the use of the solid elements, albeit unsubstantiated. The writers have been using the hexa 27 element for shell problems and have not come across any problems.

3. Case Study 1: The Stress Analysis of a Barrel Vault using Shell Elements

In our first test case, we consider the problem of the barrel-vault, a much studied shell problem [6, 11, 12]. Because of symmetry, we only need to consider a quarter of the shell. The shell has no edge beam and is supported by a diaphragm at its ends. The shell properties are: Young's Modulus = $3.0\text{x}10^6$ psi, and Poisson Ratio = 0. The shell mass density is 90 lbs/ft^2. The shell has a radius to thickness ratio of 100, and can be considered a thin shell. The shell is shown in Fig. 3.

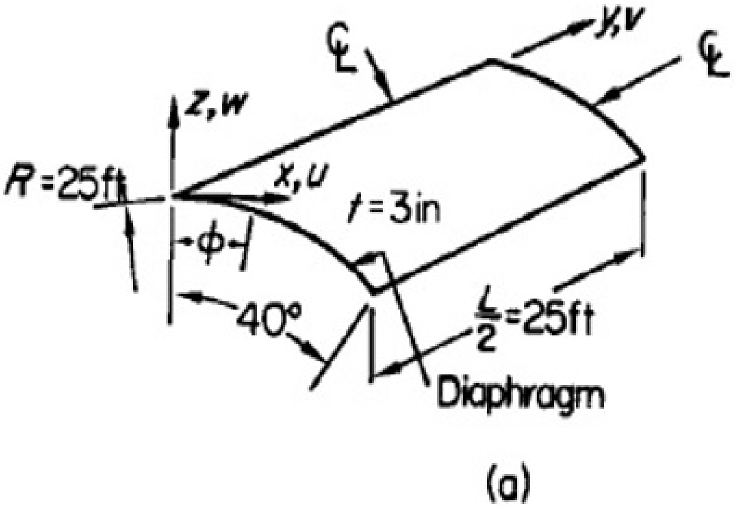

Fig. 3. Geometry of a Barrel-Vault (after [6]).

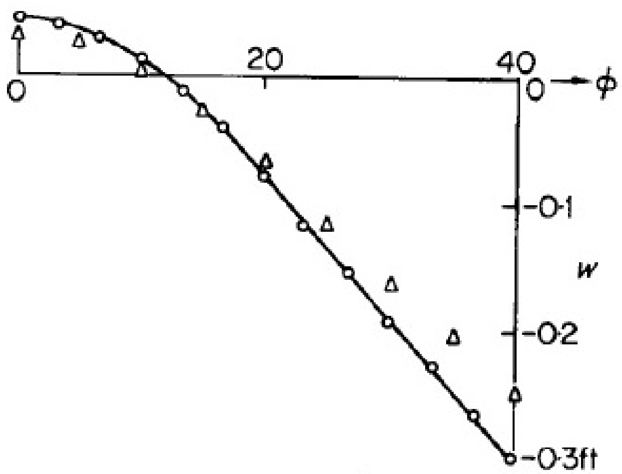

Fig. 4. Vertical displacement of mid-plane (after [6]).

The vertical displacement on the mid-plane from [6] is shown above in Fig. 4. The results agree between the Hexa-27 and the cubic serendipity elements respectively. The shell element results are stiffer.

The results for hexa 6 X 6 sits exactly over the results shown here for the 6 X 4 in [6]. The important dimension for both elements being in the mid-plane. The following table shows the variation of the maximum displacement with respect to the number of subdivisions along the mid-plane, given as the second dimension.

The analysis also included the estimation of the buckling load at the end of the incremental analysis. This buckling is indicated with a negative multiplier which indicates that the shell is more susceptible to a negative weight generated perhaps by wind forces rather than by its weight [12]. We concluded our study of this case by reducing using a 6 X 6 mesh to show that the same results as the 9 X 6 mesh could be obtained with a 6 X 6 mesh, since in general, one could not tell the preference of one dimension versus the other. Finally we reduced the shell thickness to 0.25 in. resulting in a radius to shell thickness ratio of 1200, which may be considered as a very thin shell. This is typical of the shell used in our study of the micro-lattices.

Table 1. Effect of mesh size on results second dimension is along the mid-plane.

Shell Subdivision	2 X 2	6 X 4	9 X 6
Vertical Disp. In mid-plane, in.	-0.98	-3.03	-3.54
Buckling multiplier	-13.56	-5.77	-4.53

Now the maximum vertical displacement is -16.4 ins. And the buckling multiplier becomes -0.21 respectively. A much weaker shell in resisting buckling.

4. Case Study 2: Analysis of a Pipe Elbow with a Longitudinal Crack using Solid Elements

In our second case study, we introduce, by way of two companion papers [7, 8], two new methods of estimating finite element solutions in order to demonstrate that the Hexa-27 element is the “best” choice among all available element types.

As introduced earlier in this paper and explained in full in a companion paper [7], the first new method is a nonlinear least square (NL-LSQ) fit of a series of finite element solutions of increasing mesh densities using a 4-parameter logistic distribution as the fitting function such that its upper bound parameter serves as the estimate of the finite element solution at infinite degrees of freedom.

The second method, also introduced earlier in this paper and explained in full in a second companion paper [8], is a super-parametric finite element simulation method, where not only the geometric, loading, constraint, and material property constants are parametrized, but also the mesh density, the element type, and the FEM solution platform (e.g., ABAQUS, ANSYS, LS-DYNA, et al.). This new method is very powerful in the sense that it allows a user to compare and estimate the uncertainty of a huge collection of finite element solutions for a specific problem.

To illustrate both methods, we consider the elastic deformation of a 90-degree pipe elbow with a longitudinal surface crack in one of the two weldments connecting the elbow with two straight pipes (see Figs. 5 and 6).

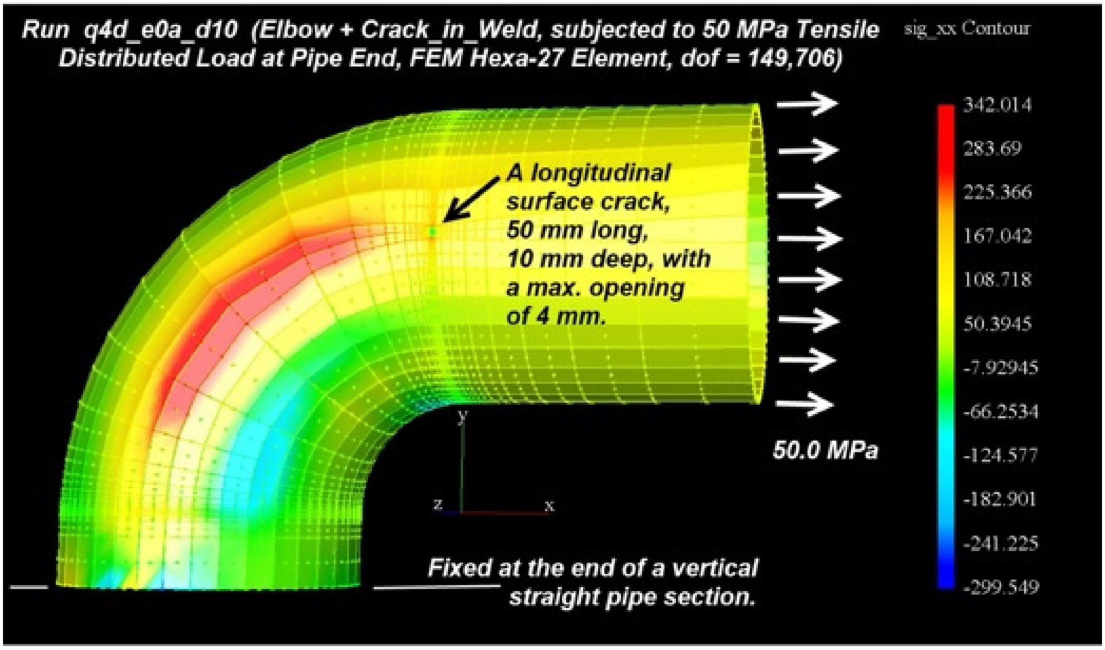

Fig. 5. Finite Element Solution using Hexa-27 elements for the Elastic deformation of a pipe-elbow with a longitudinal surface crack in one of its two weldments [7, 8].

In Fig. 7, we show the results of solving the pipe-elbow-weldment-with-crack problem using ABAQUS Hexa-08 elements for 12 mesh densities, and fitting the solutions using the Logistic NL-LSQ fit method in four steps. As the number of points increases in a plot of the crack-tip stress, S_{xx} , vs. Log_{10} (degrees of freedom), the upper bound parameter of each fit increases until it converges to a stable value, 231.7 +/- 2.0 MPa. For a complete explanation of this result, see our first companion paper [7]. In Fig. 8, we show that by using a super-parametric method (see our second companion paper [8]), we can solve the elastic deformation of the pipe-elbow-weldment-with-crack problem for more than one element type and solution platform such that the superiority of the Hexa-27 element type is clearly demonstrated.

5. Case Study 3: Analysis of the Elastic Bending of a Cantilever Beam using Solid Elements

To validate this two-method approach, i.e., a Logistic Nonlinear Least Square Fit Extrapolation method [7] and a Super-Parametric FEM Coding technique [8], we apply those two methods to a simple problem with a known exact solution, namely, the elastic bending of an isotropic cantilever beam with an end load. The problem is stated in Fig. 9, and the solution is given in Fig. 10. Knowing that the exact solution for the maximum stress at the support is 1500 MPa, we choose to compare the FEM solutions of the same problem using different element types and solution platforms, first at at 10,000 degrees of freedom, and then at an infinite degrees of freedom. It is interesting to observe in Fig. 10 that at 10,000 deg. of freedom, the solutions of the Hexa-27, Hexa-08, and Tetra-04 are quite far apart, i.e., 1375, 745, and 500 MPa, respectively, whereas at infinite deg. of freedom, they are reasonably close to the exact solution, i.e., 1560, 1450, and 1350, respectively.

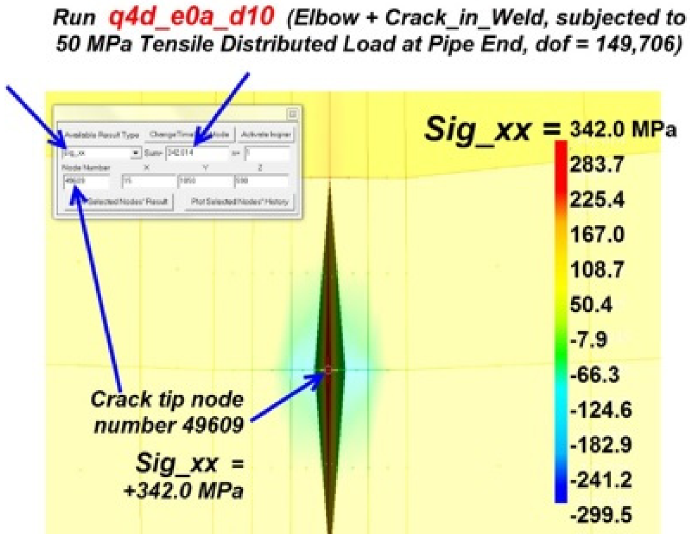

Fig. 6. A close-up of the Hexa-27 finite element soluton of the elbow+crack problem [7, 8].

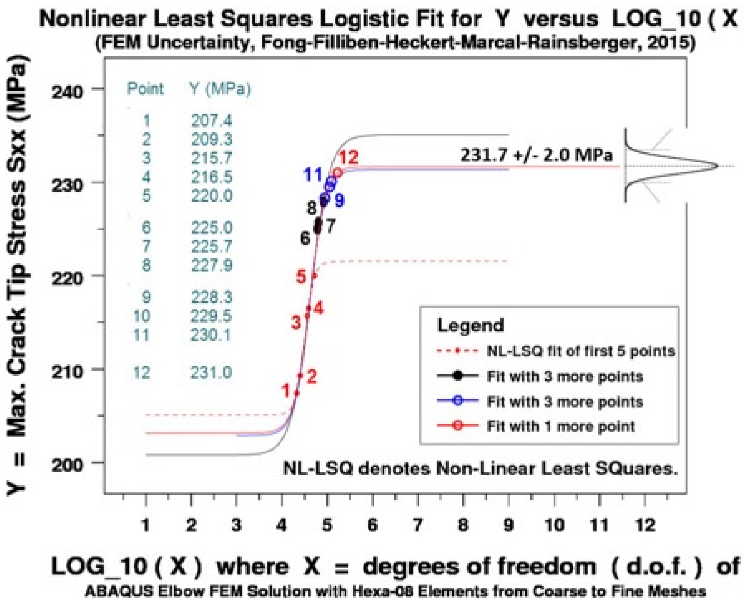

Fig. 7. ABAQUS Elbow Solutions with Hexa-08 Elements from Coarse to Fine Meshes [7].

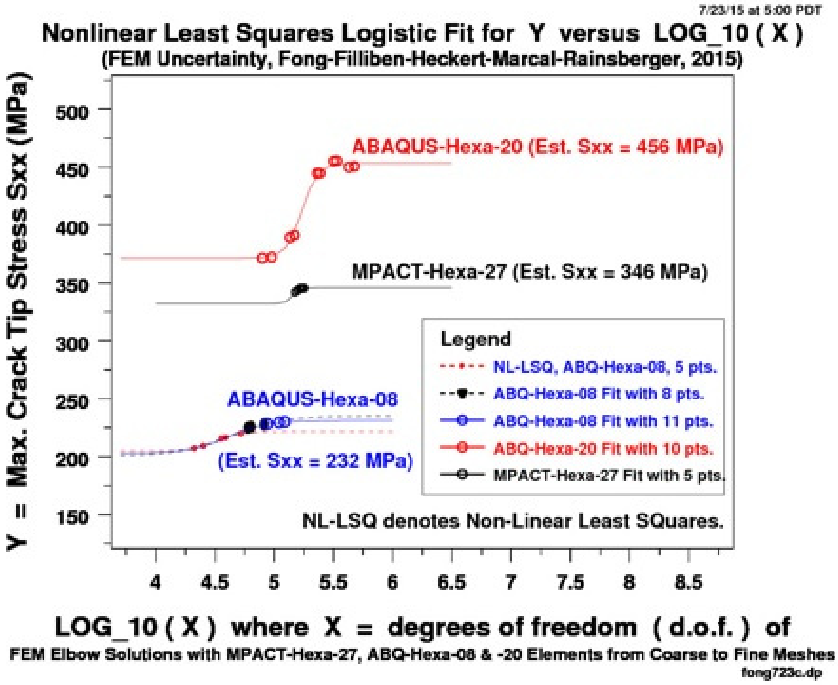

Fig. 8. FEM Elbow Solutions with Hexa-27, Hexa-20, and Hexa-08 Elements [7,8].

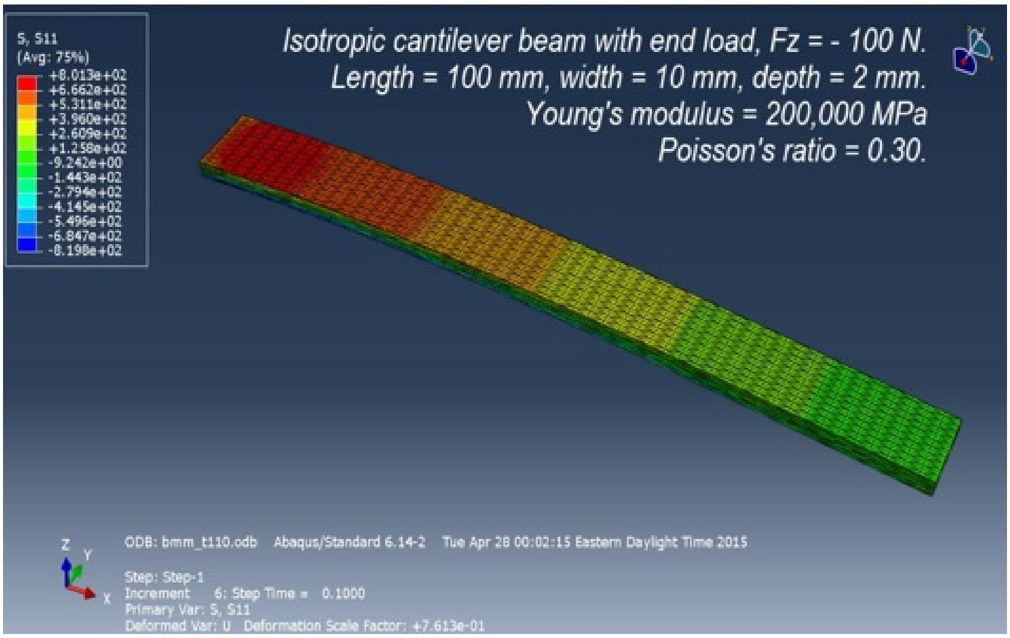

Fig. 9. FEM Solution of Elastic Bending of an Isotropic Cantilever Beam with End Load [7, 8].

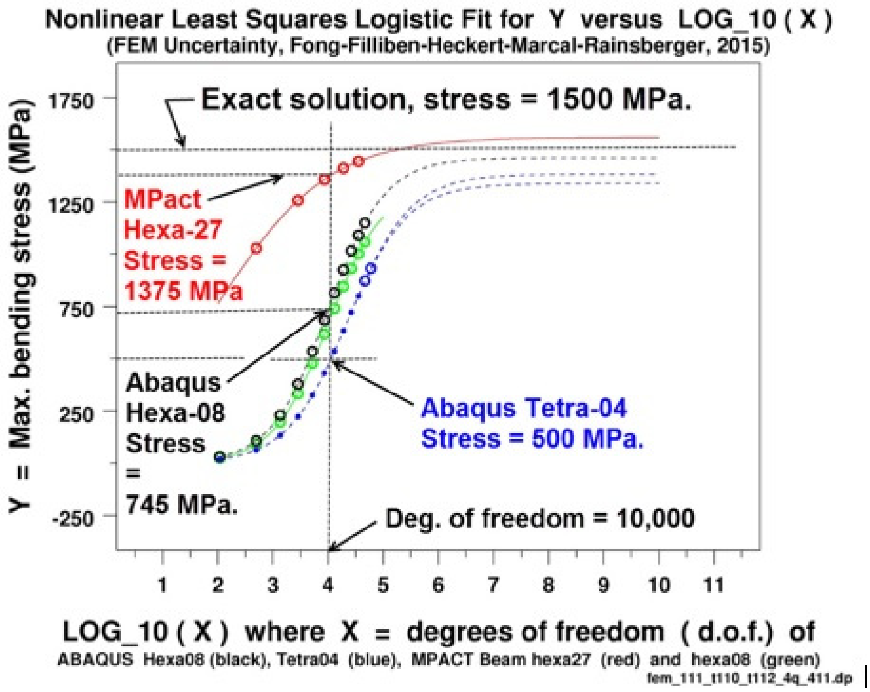

Fig. 10. Hexa-27, Hexa-08, and Tetra-04 FEM Solutions of Elastic Bending of an Isotropic Cantilever Beam under End Load [7, 8].

6. Significance and Limitations of a Hexa-27-Element-based Finite Element Analysis

The demonstration of the superiority of the Hexa-27 element in the above three case studies is significant in at least three specific areas where the accuracy and uncertainty estimate of a finite element simulation is of critical importance. The three areas of interest are:

Area-1: Public Safety.
An over-estimate of the service life of a component based on an inaccurately low stress estimate will lead to pre-mature failure and endanger public safety.

Area-2: High Cost of Maintenance.
An over-estimate of the critical stress will lead to an over-conservative maintenance schedule that will be unnecessarily costly.

Area-3: Poor Design
An inaccurate estimate of stresses or displacements inevitably lead to poor design.

The proposed two-method approach clearly has its limitations, namely, it is an extrapolation method Based on empirical numerical solution evidence, and needs additional rigorous mathematical proofs. The demonstration so far uses only elastic analysis and needs to be extended to nonlinear problems.

7. Concluding Remarks

We show in this and two companion papers [7, 8] that, when the accuracy and uncertainty estimate of a finite element simulation are critical, the Hexa-27 element with its full quadratic representation is far superior than any other commonly available element types. By introducing a logistic nonlinear least square fit method [7] and a super-parametric FEM coding scheme [8], we have developed a new approach to finite element analysis having a potential to lead to a protocol for an accurate verification of any specific finite element simulation.

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