Stress Intensity Factor Solutions for Narrow Plates

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Abstract. Accurate quantification of crack tip stress intensity values is paramount in the analysis of damage tolerant structures. The present analytical investigation seeks to determine the stress intensity solutions for crack geometries outside the existing valid solution space and expand the analyst's ability to capture representative crack growth behavior. The focus of this investigation is to calculate the stress intensity factors of single quarter-elliptical corner cracks emanating from centrally located holes in finite width plates under various loading conditions (remote tension, bending, and pin loading). Many of the available finite width corrections are singled valued and universally applied to all locations along the crack front. Early investigations into the validity of this application indicated that this correction procedure produces stress intensity values +/- 30% from new solutions. The crack depth to length ratio and depth to thickness ratio can also significantly influence the accuracy of historical finite width solutions and corrections. The analytical investigation utilizes the three dimensional virtual crack closure technique and well-structured, completely hexahedral, element meshes. Stress intensity values are generated for a wide range of ratios for crack depth to crack length, crack depth to sheet thickness, hole radius to sheet thickness, and sheet width to hole diameter. This effort is being executed under a DoD Technical Corrosion Collaboration program.

Introduction

Many damage tolerance analysis codes [1,2,3] use the readily available Newman-Raju [4] semi-empirical stress intensity factor (SIF) equations for part-through cracks emanating from a centrally located hole in a flat plate. These solutions were developed by equation fits to a relatively small amount of finite element analysis (FEA) cracking scenarios; and are typically employed for their ease of use in repetitive computations typical in damage tolerance analyses (DTA). However, with the continual advancement in the computational speed of analysis systems, the DTA community is ever more reliant upon more exact simulations, which can be obtained using a more robust cracking scenario data set.

Available Solutions

The Newman-Raju solutions were developed for a range of through-thickness crack depth, a, to crack length in the transverse direction, c, ratios (or crack aspect ratios) of $0.2 \leq a/c \leq 2.0$, a range of hole radius, R, to plate thickness, t, ratios of $0.5 \leq R/t \leq 2.0$. Furthermore, limitations on the crack length are such that the combined crack length and hole radius should be less than one-quarter of the plate width, W, (i.e. $(R + c) < W/4$). Basic plate and crack geometry is identified in Fig. 1. These solutions are further corrected for the effect of the plate's finite width. These single value corrections are calculated for a given crack and plate geometry and multiplied to the SIF at every location along the crack front. From the current investigation, it is clear that the semi-empirical equation fits and single-valued finite width correction can produce significant errors in the calculated SIF versus the actual crack drive forces in actual structure.
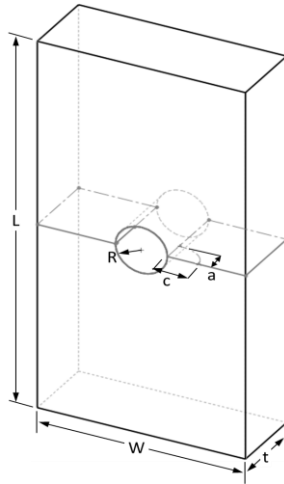
Figure 1. Plate and crack geometry definitions

Solution Space

Not only does the current effort seek to increase the accuracy of SIF calculations within the Newman-Raju data space, but aims to also greatly expand the solution space to preclude solution extrapolation. The present investigation includes uniaxial tension, out-of-plane bending, and bearing (pin) load cases (see Fig. 2). The range of crack and plate geometries is extensive. The large solution space covers the following geometries for single corner cracks (SCC):

$1.1 \leq W/D \leq 20$ $0.2 \leq D/t \leq 20$ $0.1 \leq a/t \leq 0.99$
$0.1 \leq a/c \leq 10$ $L/W = 2.0$

where D is the hole diameter, and L is the plate length. These plate geometries cover many of the short edge distance to hole diameter ratios commonly found in aerospace structures.
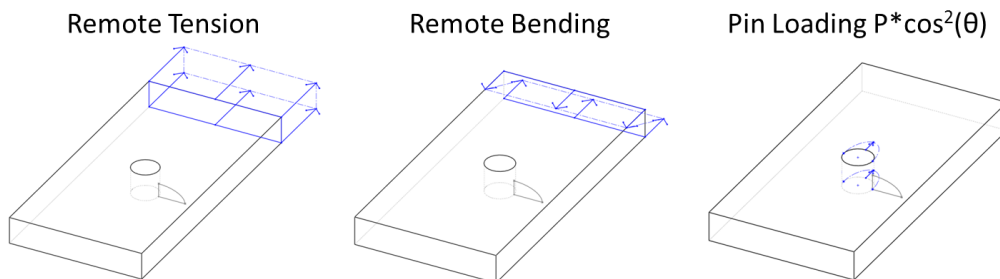

Figure 2. Load cases investigated herein

Solution Process

Well-structured, fully hexahedral, finite element (FE) cracked half-plate models, similar to the one seen in Fig. 3, are automatically generated and interrogated for mesh quality. Over 150,000 individual crack models are used to generate a sufficiently high-fidelity solution space as to minimize interpolation error in SIF extraction at intermediate crack and plate geometries.

The FE models are generated by a multi-step process aimed to reduce the total number of degrees-of-freedom (DoF), which is a major driver in the time to solution. Maintaining a sufficiently dense mesh to minimize the discretization and numerical error inherent in FE analyses and ensure the applicability of the virtual crack closure technique (VCCT) [5] is also a major priority. Two of the main drivers of the mesh element sizes along the various crack fronts are: 1) the proximity of the crack tip to free surfaces; and 2) the curvature of the crack tip. All of the mesh density data is defined through several MATLAB subroutines [6] which creates the input files for mesh and boundary condition application in the preprocessor, TrueGrid [7]. Many other typical "rules-of-thumb" checks are performed to ensure mesh quality prior to solution submittal (e.g. element aspect ratio, skew, and Jacobian checks).

After the mesh generation the mesh geometry and boundary condition files are uploaded onto one of several Department of Defense (DoD) high performance computing (HPC) centers for execution. The FE analysis solver used thus far in the project is ZIP3D [8].

After successful execution of the solver, the results files are interrogated in order to extract the needed crack front nodal forces and crack wake nodal displacements used in the determination of the strain energy release rate, G, and subsequently the SIF, as presented in [5], along the entire crack front.
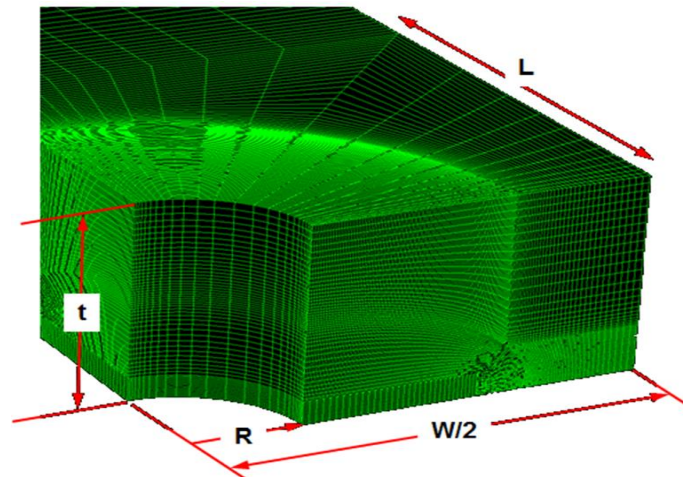

Figure 3. One quadrant of a plate FE model showing the crack plane

Model Validation

Multiple models were generated at the onset of the program to determine the mesh density parameters needed to ensure acceptable accuracy in the calculated SIFs. Several different model results are presented as a function of the normalized parametric angle, Φ, as defined in Fig. 4. These models are all loaded in uniform tension with the geometry values shown within Fig. 5, and three (3) different crack aspect ratios are investigated. Each crack aspect ratio is plotted with three (3) levels of mesh density along the crack front.

Note that all of the mesh density cases lay nearly perfectly on top of one another for each crack aspect ratio. The only significant differences occur at the crack vertices, where the crack front intersects the free surfaces. It appears that the coarsest mesh still adequately captures the variation in SIF along the crack front and completely contains the boundary layer effect at the free surfaces, where the SIF is, by definition, zero, within the vertex elements.

Comparisons to available industry results were also performed for several cases. The results are shown in Fig. 6 for several double symmetrical corner crack (DSCC) cases. (Note that these cases where chosen to reduce the number of DoF and to avoid use of the Shah correction factor typically employed to extract single corner crack SIFs from double corner crack solutions [9].) Results of the Newman-Raju solutions [4], the AFGROW Advanced solutions [1], which employ the Fawaz-Andersson [10] solutions along with an additional correction factor in addition to the Newman-Raju correction, and the current method's solutions are presented in Fig. 6.
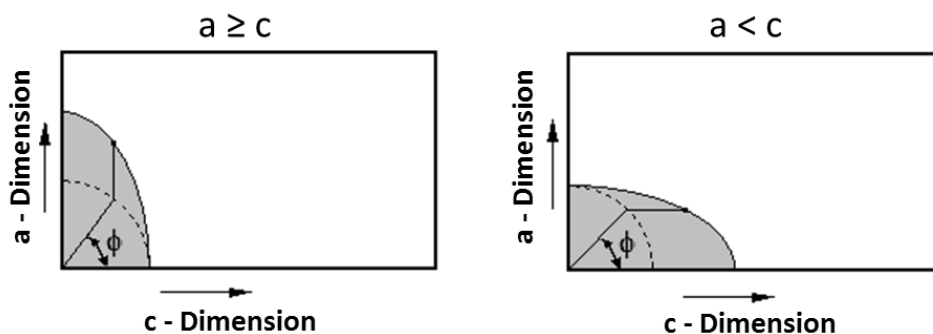

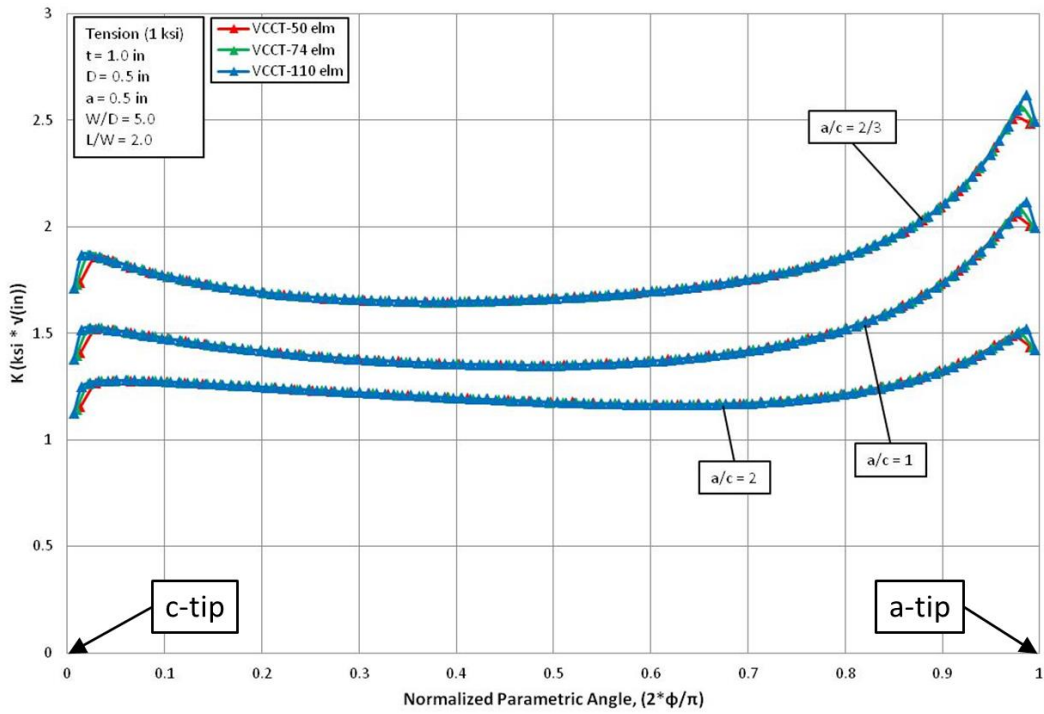Figure 4. Parametric angle definition

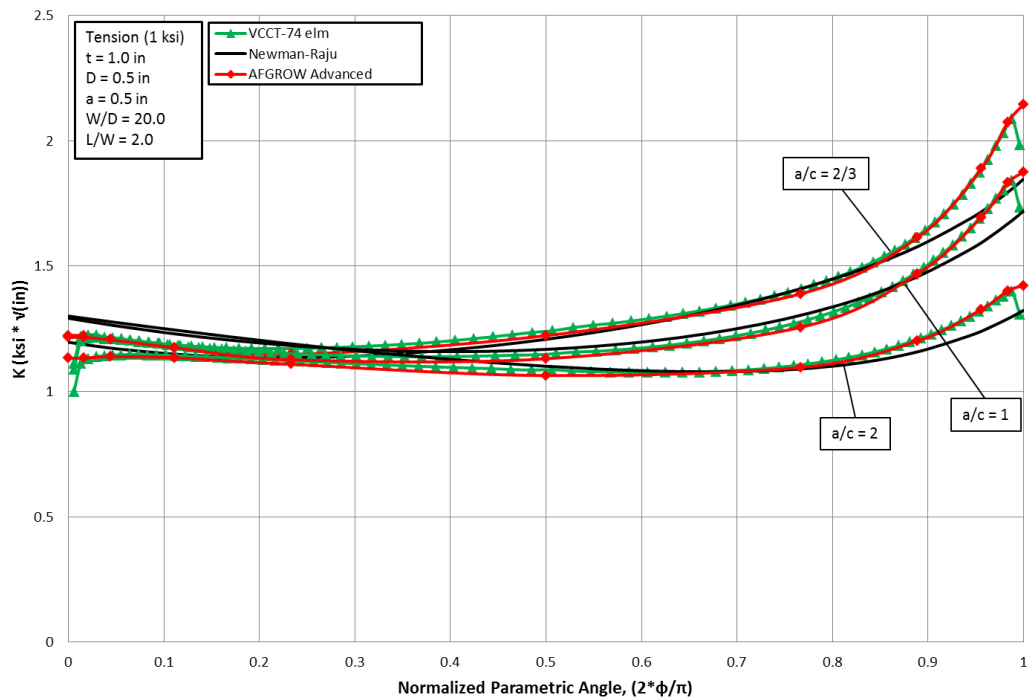Figure 5. Mesh density determination models' results

Figure 6. Comparison of SIF solutions for several DSCCs

As can be seen in Fig. 6, there is excellent agreement between the AFGROW Advanced solutions and the current work for the given geometry. However, there is a large difference in the Newman-Raju solutions when compared to the others for these likely crack scenarios, especially near the crack vertices. It should be noted that the vertex behavior displayed in Fig. 6 for the AFGROW advanced solutions is more pronounced in the raw data sets than the data displayed here, which are curve fits used in the DTA simulations [11].

Results

Because there are over 150,000 different plate and crack geometry models investigated in this effort, only a small snapshot of the analysis results are presented here.

The *most* dramatic differences between the results of the present investigation and those available in industry are expected to exist at the extreme crack and plate geometries (e.g. very large or small crack aspect ratios, significant portions of the crack plane occupied by a crack, crack tip(s) near the free surface(s)). Therefore, in order to gain further confidence in the new numerical results, several comparisons are made with existing industry solutions. Note that only AFGROW Advanced solutions are presented here, as much of the geometry presented is well outside of the reported solution space of the Newman-Raju equations.

Fig. 7 shows several cracking scenarios with an aspect ratio of $a/c = 1.0$, with decreasing plate widths. As can be seen from the figure, good agreement is seen between the AFGROW Advanced solutions and the present investigation's results when the plates are relatively wide. However, as the plates begin to narrow, changes are seen; especially near the crack's c-tip. Note that the differences in the solutions are much greater at the crack's c-tip for the narrowest plate. This is mainly due to the encroachment of the crack's c-tip on the free surface along the side of the plate.
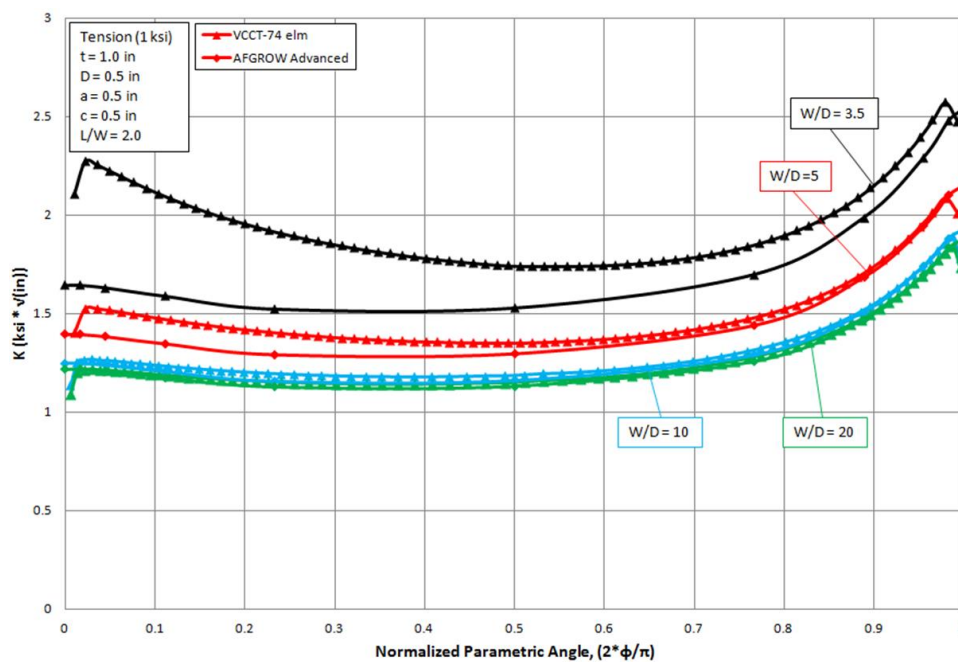

Figure 7. Results comparisons with $a/c = 1.0$

Impact

The large differences between the commercially available SIFs for corner cracks emanating from centrally located holes in finite width plates and the results of the present investigation can have a significant effect on the structural integrity community. Typical inspection protocols within the aerospace community set initial and recurring inspection intervals based upon the results of DTAs. As an example of the potentially drastic differences in the life predictions, an example is presented in Fig. 8 for a laboratory corrosion/fatigue specimen with a significant finite width effect. An aircraft wing, tension only, block spectrum is applied in the simulation. The geometry of the part is detailed below.

$t = 0.126\text{in}$	$R/t = 0.9375$	$W/D = 1.667$
$a_0/t = 0.1$	$a_0/c_0 = 1.0$	$L/W = 2.0$

Note that the "Classic" curve in Fig. 8 is the AFGROW implementation of the Newman-Raju solutions, and the "Advanced-2pt" are from the AFGROW Advanced module. The "VCCT" curve is developed by using data taken from the present investigation and formatted into an AFGROW usable format for user-defined SIF life predictions. As can be seen in Fig. 8, a non-arbitrary difference in the DTA life predictions is visible.
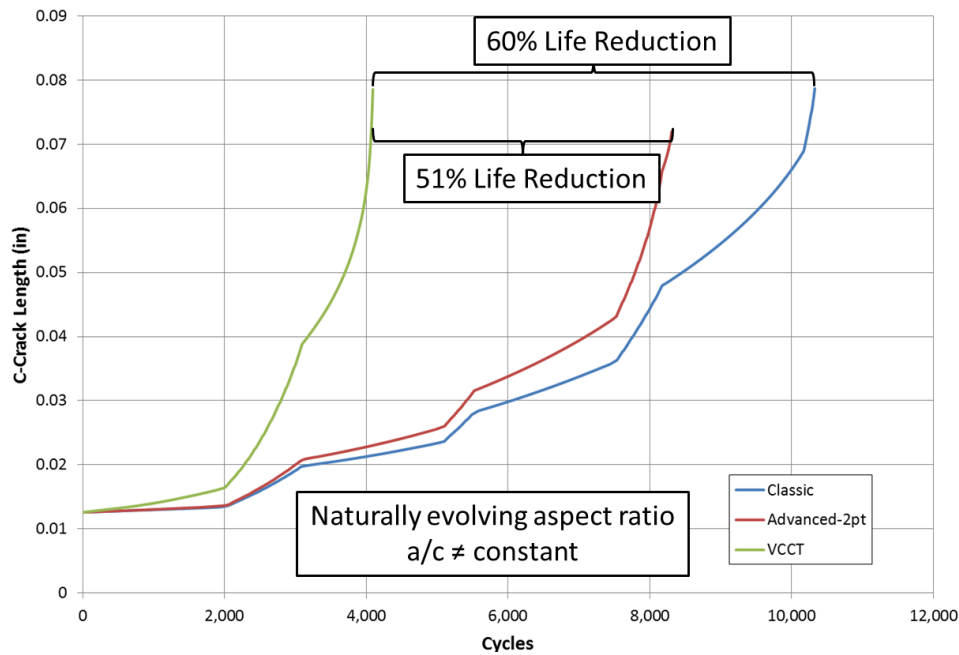
Figure 8. DTA life prediction comparisons

Conclusions

The aim of the present investigation is to develop a large and dense solution space of highly accurate SIFs for corner cracks emanating from centrally located holes in finite width plates under various loading conditions. It has been shown that commercially available SIF solutions can have significant errors embedded, especially for extreme crack and plate geometries. These non-trivial differences in SIF solutions can lead to significant changes in aerospace inspection intervals and potentially unsafe conditions if not addressed within an appropriate time frame. It should also be noted that many crack and plate geometries investigated here have produced SIF values which are smaller than those compared to within this effort. The primary objective here is to produce accurate SIF data for the DTA community at-large.

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